Lesson 7-4: Areas of Trapezoids, Rhombuses & Kites

A little bit tougher…

Once you knew how to find the area of a triangle and rectangle, finding the area of a parallelogram wasn't that tough. Today we're going to work with a few more complex figures: trapezoids, rhombuses and kites. These are tougher because they aren't necessarily regular figures.

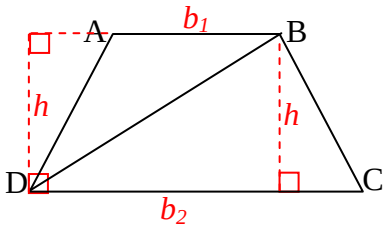
A visual solution

If you look in the text you will see visual solutions for the area formula for these figures. If you haven't already, take a few minutes and review them. They are very instructive. We tend to think very algebraically and learning to see things geometrically is a great skill to develop.

A definition…

Before we dive in, we need to make sure we're clear about a term. The height of a trapezoid is the length of a segment perpendicular to the bases of the trapezoid.

Algebraic solution for the area of a trapezoid



If you take a trapezoid *ABCD*, you can divide it into two triangles: *ABD* and *BCD*. If you take the bases of the trapezoid as the base of the respective triangle, both triangles have the same height as the trapezoid. The area of $\Delta ABD = \frac{1}{2} b_1 h$ and $\Delta BCD = \frac{1}{2} b_2 h$. The area of the trapezoid is the sum of the areas of these two triangles or:

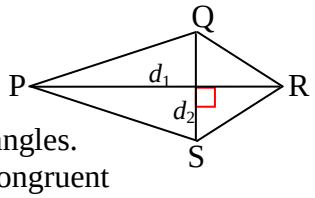
$$area\ ABCD = \frac{1}{2} b_1 h + \frac{1}{2} b_2 h = \frac{1}{2} h(b_1 + b_2)$$

Theorem 7-10 Area of a Trapezoid

The area of a trapezoid is half the product of the height and the sum of the bases.

$$A = \frac{1}{2} h(b_1 + b_2)$$

Algebraic solution for the area of a kite (or rhombus)



Given a kite *PQRS*, you can immediately see the diagonals form four triangles. You can also consider each diagonal dividing the kite into two triangles. The diagonal that splits the congruent sides (in this figure, *PR*) creates two congruent triangles: triangles: $\Delta PQR \cong \Delta PSR$. Since they are congruent, their heights are equal which can be used to show that *PR* bisects *QS*. Thus the height of each triangle is ½d_2. The area of the kite is the sum of the areas of the triangles, so:

$$area\ PQRS = \frac{1}{2} d_1 h_1 + \frac{1}{2} d_1 h_2 = \frac{1}{2} d_1 (h_1 + h_2) = \frac{1}{2} d_1 (\frac{1}{2} d_2 + \frac{1}{2} d_2) = \frac{1}{2} d_1 d_2$$

Theorem 7-11 Area of a Rhombus or a Kite

The area of a rhombus or a kite is half the product of the diagonals.

$$A = \frac{1}{2} d_1 d_2$$

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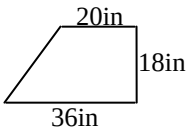
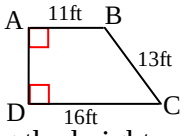
Examples

1. A car window is shaped like the trapezoid shown. Find the area of the window.

$$area = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}\cdot 18\cdot(20+36) = \frac{1}{2}\cdot 18\cdot 56 = 504\,in^2$$



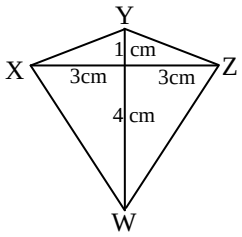
2. Find the area of trapezoid *ABCD*.



Side *AD* is the height. Constructing the height segment at vertex *B* forms a right triangle with base 5 *ft* (16 – 11). Using the Pythagorean Theorem we find that $h^2 + 5^2 = 13^2; h = AD = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$.

$$area\ ABCD = \frac{1}{2}\bullet 12\bullet(11+16) = 6\bullet 27 = 162\ ft^2$$

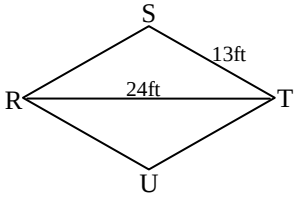
3. Find the area of kite *XYZW*.



$$d_1 = 1+4 = 5;\ d_2 = 3+3 = 6$$

$$area\ XYZW = \frac{1}{2}\bullet 5\bullet 6 = 15 cm^2$$

4. Find the area of rhombus *RSTU*.



Diagonal *RT* is the perpendicular bisector of diagonal *SU*. Together they form four right triangles with base 12, hypotenuse 13 and height *h* (½ *SU*). Thus

$h^2 + 12^2 = 13^2; h = \sqrt{13^2 - 12^2} = \sqrt{25} = 5;\ SU = 2h = 10\ ft$.

$$area\ RSTU = \frac{1}{2}\bullet 24\bullet 10 = 120\ ft^2$$

Homework Assignment

p. 376 #1-21, 25-37 odd